



University
of Victoria

Graduate Studies

Notice of the Final Oral Examination
for the Degree of Doctor of Philosophy

of

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**“A Numerical Study of the Comparison Between Convectively Forced
Hydrostatic and Non-hydrostatic Mesoscale Processes”**

Department of Mathematics and Statistics

Friday, December 9, 2016
9:30 A.M.
David Turpin Building
Room A144

Supervisory Committee:

Dr. Slim Ibrahim, Department of Mathematics and Statistics, University of Victoria (Supervisor)
Dr. Boualem Khouider, Department of Mathematics and Statistics, UVic (Member)
Dr. Adam Monahan, School of Earth and Ocean Sciences, UVic (Outside Member)

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Dr. Tao Lu, Department of Electrical Engineering, UVic

Abstract

Mesoscale processes in the atmosphere refer to the atmospheric processes that take place within a scale of 20 - 2000 kilometres. Atmospheric phenomenon like thunderstorms, inertia-gravity waves, jet streaks, fronts and many others have a length scale within the range of 20 - 2000 km and hence are included in the category of Mesoscale dynamics. In the study of these processes, because the horizontal length scales are very large as compared to the vertical scales, often vertical accelerations are ignored. Such type of processes are termed as hydrostatic mesoscale processes. If the vertical accelerations are not ignored, then the mesoscale processes are known as nonhydrostatic mesoscale processes. This research work gives a study of the convectively forced nonhydrostatic mesoscale processes. Comparison is made between the results of both hydrostatic and nonhydrostatic mesoscale processes. To do so, a stably stratified, two-dimensional, Boussinesq, nonrotating, inviscid uid experiencing a thermal forcing is considered under both hydrostatic and nonhydrostatic assumptions. While explicit analytic solutions are available for the hydrostatic cases under both a constant and a shear (linear in z) background profile, to understand the nonhydrostatic cases, a complete discretization of the governing linearized set of equations is carried out for the same background profiles. It has been found that the hydrostatic assumption does not depict the complete dynamics of the process. A horizontal propagation of a wave which is found to be present in the nonhydrostatic cases, is completely missing in the hydrostatic cases.

Further, we show that for both, hydrostatic and nonhydrostatic cases, a sinusoidal shear background profile is nonlinearly unstable. However, because of mathematical difficulties, this work is done for a more specific convectively forced mesoscale processes. More specifically, a sinusoidal background profile is chosen and the external forcing is also treated in a more specific manner. Different from the study of flows forced by an external heating source, where the impacts of the forced wave modes with the atmosphere are studied, for various processes we need to allow the feedback of the atmosphere to the latent heating and a well known way to get such a

feedback of the atmosphere is to assume that the diabatic heating is everywhere proportional to the vertical velocity. This kind of treatment of the external forcing is appropriate, for instance, for the processes like moist convection. Under such an assumption, the heating will respond to the motion of an air parcel. If the parcel rises upward, latent heat will be released and evaporative cooling will be observed if the parcel of air undergoes a downward motion. To prove the nonlinear instability for a sinusoidal background profile, first the well-posedness of the governing set of nonlinear equations is established. Then, a linear unstable mode is constructed using a method of continued fractions and then finally, following Grenier's idea, it is shown that the constructed linear unstable mode is also nonlinearly unstable.